Implementation of asymmetric tooth root geometry for downsizing automotive transmission gears

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Summary

This article proposes a design method for improving the tooth root breakage strength of automotive transmission gears. This method for improving tooth root breakage strength involves increasing the tooth root radius of curvature so as to relax tooth root stress. The tooth root radius of curvature is limited by the symmetric geometry of the cutting tool tip radius for both the driving- and coasting-side tooth flanks.

In this study, a method is proposed for designing gears with an asymmetric tooth root geometry along with a method for designing cutting tools for machining such gears with stable quality. The effectiveness of the proposed methods was validated, and it was confirmed that gear service life can be lengthened against tooth root breakage. The results verified that the implementation of the proposed design methods is effective for improving tooth root breakage strength and extending gear service life.

1. Study background

There have been demands in recent years for further reduction of automotive transmission size and weight from the standpoints of improving vehicle fuel economy and ensuring collision safety, among other requirements. As a result, the durability and reliability of transmission gears must be ensured under even more severe operating environment conditions.

This article focuses on tooth root breakage strength, as breakage is a principal failure mode of automotive transmission gears. A new method of designing gears is proposed that can improve tooth root breakage strength. The traditional approach to designing tooth root breakage strength has been to use S-N diagrams of tooth root fatigue breakage. The modified Miner's rule is used to define the target number of load cycles for gears based on the input torque and input frequency that occur in real-world driving situations (referred to here as the field loading frequency). Tooth root bending stress is defined as the criterion for satisfying this target performance. Gear specifications are then designed so as to satisfy this criterion.

In order to improve tooth root breakage strength, tooth root bending stress must be reduced. A key factor in this regard is to relax the stress concentration by increasing the tooth root radius of curvature. The size of the tooth root radius of curvature is limited by the conditions needed for a viable hob as the gear tooth cutting tool. This limitation stems from the symmetric geometry of the hob cutting edge radius for both the driving- and coasting-side tooth flanks. Moreover, because the field loading frequency is larger on the driving side than on the coasting side, the gear size is determined by the tooth root breakage strength required by the driving-side tooth flanks. Accordingly, it was reasoned that gears could be downsized if they could be designed with an asymmetric tooth root radius of curvature, that is, by designing the driving-side and coasting-side tooth flanks with a large and small radius of curvature, respectively.

Therefore, this article proposes a method of designing gears with an asymmetric tooth root geometry and a method of designing hobs for machining such gears with stable quality in mass production. The validity of the proposed design methods has been verified by conducting durability tests on CVT units built with gears designed and produced using these methods. The test results

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confirmed that the tooth root breakage life of the tested gears was lengthened. Finally, the implementation of mass production of gears having an asymmetric tooth root radius of curvature will be described.

2. Concept of strength design for an asymmetric tooth root radius of curvature (R)

As mentioned above, it is well known that tooth root bending stress is a key factor of tooth root breakage strength. The authors applied the method proposed by Kubo and Umezawa in (Fig. 1)[1] to calculate the load distribution on a simultaneous line of contact. Tooth root bending stress was then calculated from the load distribution, and tooth root breakage strength was designed on that basis.

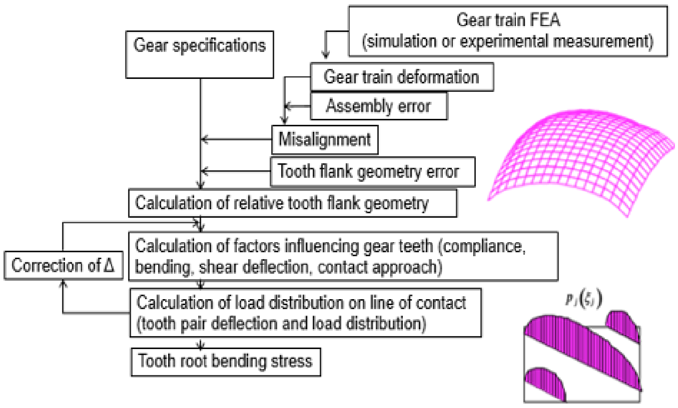


Fig. 1 Flow chart of the calculation of tooth root stress using the equation proposed by Kubo and Umezawa

The tooth root bending stress used in this calculation method can be calculated using the following expression proposed by Aida and Terauchi[2] for calculating tensile bending stress.

$$\sigma_t=\left(1+0.08\frac{S}{\rho}\right)\left(0.66\sigma_{Nb}+0.40\sqrt{\sigma_{Nb}^2+36\tau_N^2}+1.15\sigma_{Nc}\right) \quad \cdots\cdots (1)$$

where S is the tooth thickness at the position of the critical cross section, ρ is the tooth root radius of curvature, σNb is nominal bending stress, τN is nominal shear stress and σNc is compressive stress. The value of ρ in the equation must be given as a dimension that allows viable hob cutting edge geometry.

Equation (1) is an expression derived on the basis of gears having a symmetric tooth root radius of curvature. However, it was reasoned in this study that Eq. (1) was applicable even to gears having asymmetric tooth root geometry so long as tooth deflection did not change. Specifically, it was assumed that the bending stress of gears having an asymmetric tooth root radius of curvature can be calculated by making the tooth thickness S at the position of the critical cross section the same as that of conventional gears with symmetric tooth root geometry and applying the respective tooth root radius of curvature ρ of the driving- and coasting-side tooth flanks to Eq. (1).

3. Construction of a method for designing gears with asymmetric tooth root geometry

The proposed design method was constructed using the second reduction gear pair shown in Fig. 2 of a CVT for use on midsize to large passenger vehicles. The specifications of the target gear pair are listed in Table 1.

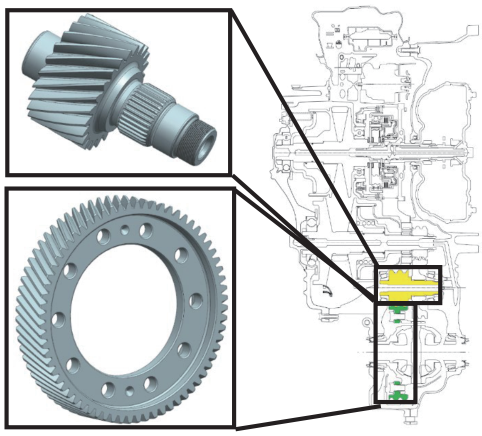


Fig. 2 Continuously variable transmission unit for medium to large passenger cars and a second reduction gear pair.

Table 1 Geometries of target gear pair

			Reduction gear	Final gear
Module	m_n	mm	2.48	←
Pressure angle	α_n	deg.	19	←
Helix angle	β	deg.	28(LH)	28(RH)
Number of teeth	z	-	23	68
Tip diameter	d_a	mm	71.9	195.3
Root diameter	d_f	mm	58.24	181.84
Facewidth	b	mm	38.9	38

Transmission gears are not used under steady torque inputs during real-world driving. Various torque levels are input to the transmission, acceleration and deceleration events are repeated, and the torque input to the gears differs between the driving side and the coasting side. An example of the field loading frequency is shown in Fig. 3.

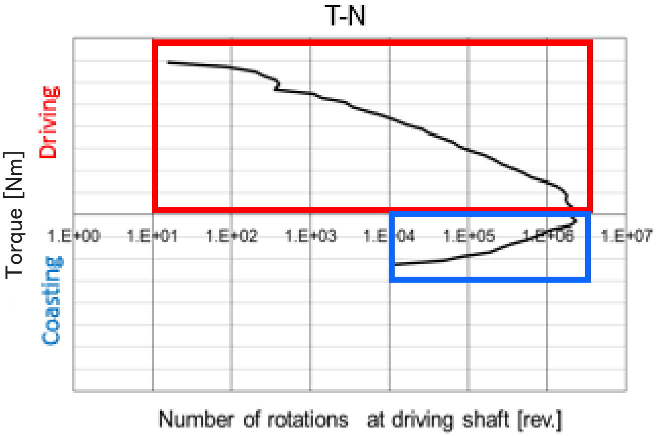


Fig. 3 Field loading frequency

The figure indicates that the predominant share of the field loading frequency is input to the driving side compared with the coasting side. Because the field loading frequency is small on the coasting side, the tooth root breakage strength of the coasting-side tooth flanks under this loading frequency is designed on the basis of the fatigue limit. Accordingly, it is not necessary for driving-side and coasting-side tooth flanks of transmission gears to possess the same tooth root breakage strength.

Therefore, the target gear pair is intentionally designed with a different size of fillet radius of curvature for the driving- and coasting-side tooth flanks, enabling the necessary strength to be secured on each side. Specifically, the gears are designed with a large radius of curvature for the driving-side tooth flanks and a small radius of curvature for the coasting-side tooth flanks. The gear tooth root radius of curvature is determined based on the field loading frequency (horizontal axis) on the driving side so that the tooth root bending stress is below the criterion; the radius of curvature for the coasting-side tooth flanks is designed based on the coasting-side field loading frequency so that the tooth root bending stress is below the fatigue limit.

Tool life and the machined accuracy of tooth flanks after the hobbing process are taken into account in designing hobs for machining gears with different radii of curvature for the driving-side and coasting-side tooth flanks. If the radius of curvature R of the hob cutting edge is excessively small, it can be expected that hob edge chipping or early crater wear may occur, resulting in a substantial reduction of tool life and increased costs. Accordingly, care is taken when manufacturing hobs to ensure the minimum necessary radius of curvature R of the hob cutting edge for coasting-side tooth flanks. In addition, in cases where the radius of curvature of the hob cutting edge differs between the driving- and coasting-side tooth flanks, care is taken so that the same amount of stock is removed from the tooth roots of the right and left tooth flanks in the tooth flank finishing process. If it is not identical, tools used in the tooth flank finishing process for gears with asymmetric tooth root geometry would be subjected to different cutting loads on their right and left cutting edges. It can be expected that this condition could cause vibration and produce undulations on machined tooth flanks. Therefore, as shown in Fig. 4, a protuberance geometry was designed for unifying the diameter at the start of undercutting following hobbing in order to ensure that stock removal on the tooth root side is identical even for gears having an asymmetric tooth root radius of curvature.

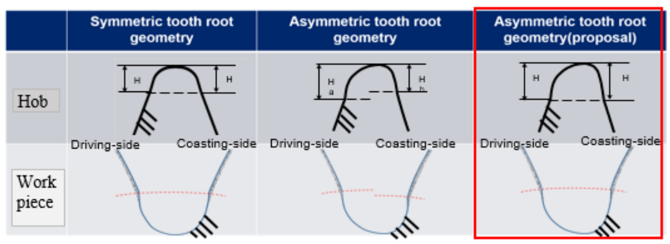


Fig. 4 Comparison of hob and workpiece

A concrete design was then executed based on the concept described above. The criterion for the tooth root bending stress of the driving-side tooth flanks was set at 1,220 MPa based on the field loading frequency of a vehicle fitted with symmetric gears, and a fatigue limit design was adopted for the coasting-side tooth flanks. To satisfy these values, the tool cutting edge was designed with radii R of 1.116 mm for the driving-side tooth flanks and 0.372 mm for the coasting-side tooth flanks.

If these tool cutting edge radii R are ensured, the desired tooth root breakage strength can be satisfied and also productivity can be secured. The hob design method was also applied to unify the diameter at the start of undercutting of the target gear pair in order to obtain stable machining accuracy in the tooth flank machining process. The specific tool dimensions and tooth root geometry are shown in Figs. 5 and 6, respectively.

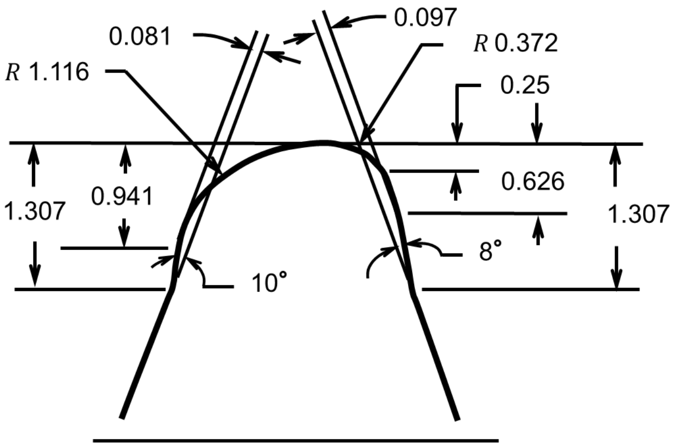


Fig. 5 The specific tool dimensions

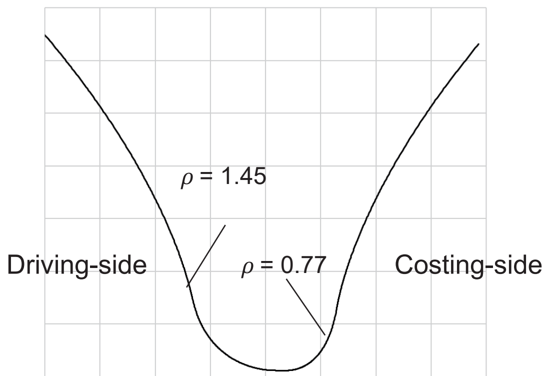


Fig. 6 Tooth root geometry

An estimation was made of the effects of applying an asymmetric tooth root radius of curvature to the target gear pair. Two types of target gear pairs were designed for the gear specifications in Table 1. One gear pair had a symmetric tooth root radius of curvature and the other pair had an asymmetric radius of curvature. The face width was designed to satisfy the criterion for tooth root bending stress. The gear volume was calculated from the difference in the face width, and the effect of the asymmetric design on improving torque density is shown conceptually in Fig. 7. The results indicate that a size reduction of 9.5% can be expected, thus confirming that an asymmetric tooth root radius of curvature can contribute significantly to downsizing transmission gears.

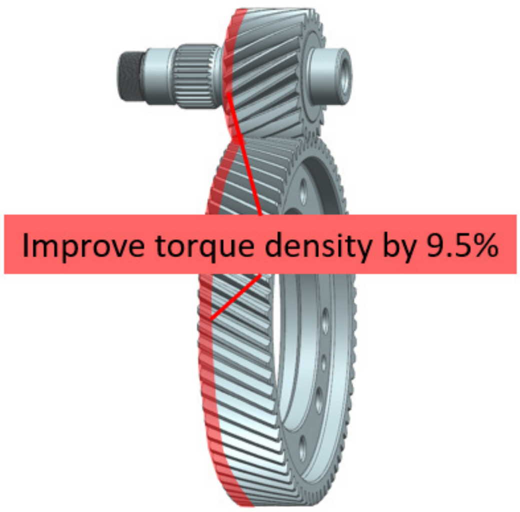


Fig. 7 Concept for improving torque density

In conducting durability tests, it is essential to know the accuracy of the gears being evaluated because that information is critical for the durability evaluation. Because this study focused on tooth root breakage strength, measurements were made of the tooth root geometry.

The tooth root geometry of the test gears was machined with hobs that were designed as explained earlier. In order to evaluate whether the tooth root geometry was machined according to the design intention, it had to be represented geometrically. The tooth root geometry was evaluated in a cross section perpendicular to the axis. Therefore, an equation was calculated for the tooth root fillet curvature in the cross section perpendicular to the gear axis based on the hob tooth profile dimensions. A simulation for calculating the tooth root geometry was created, and the theoretical geometry and the actual geometry were superimposed as shown in Fig. 8. As seen in Fig. 9, the actual and calculated tooth root geometries showed good agreement, indicating that the test gears had the desired tooth root geometry as expected.

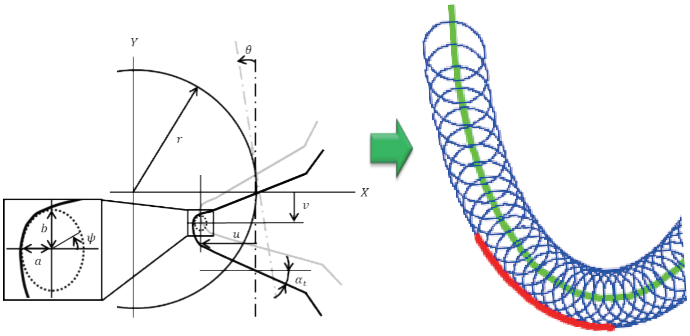


Fig. 8 Simulation for calculating the tooth root geometry in the perpendicular cross section

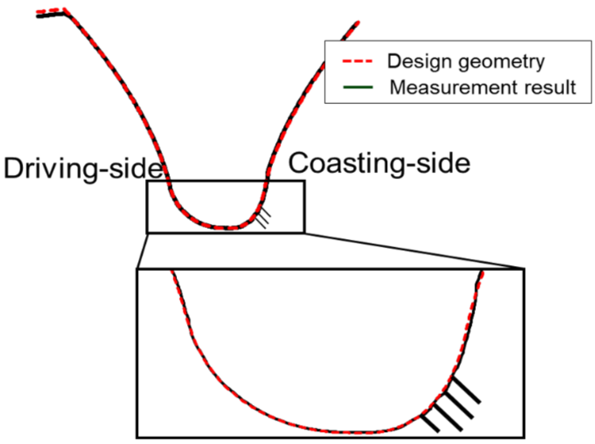
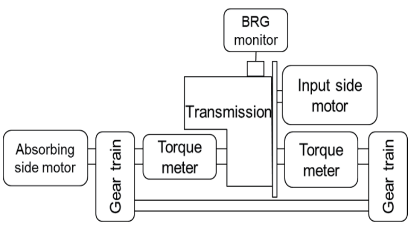


Fig. 9 Comparison of calculated and measured geometries

4. Experimental validation

Evaluations of tooth root breakage strength were conducted using the motor-dynamo testing system outlined schematically in Fig. 10. A photograph of the test stand is shown in Fig. 11.



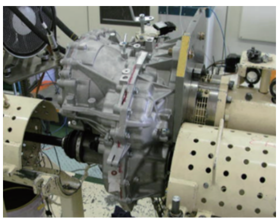


Fig. 10 Outline of test stand **Fig. 11 Photo of test stand**

The target gear pair was incorporated in a CVT unit installed on the motor-dynamo. Rotation was applied by the input-side motor and the absorbing-side motor was braked to generate the desired input torque. Durability tests were conducted under conditions of a specified rotational speed and torque until tooth root breakage occurred.

4.1 Test results

The number of cycles to tooth root breakage and the calculated tooth root stress were summarized in an S-N diagram; an example of results is presented in Fig. 12. The red circles in the diagram are for gears with an asymmetric tooth root radius of curvature and the black circles are for gears with a symmetric tooth root radius of curvature.

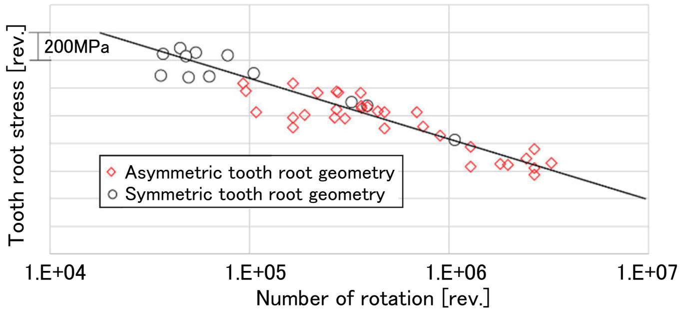


Fig. 12 S-N diagram

It is seen that the test results for both the symmetric and asymmetric tooth root geometries are plotted on the same S-N curve. Consequently, the results confirm that the method proposed for calculating tooth root stress is also applicable to gears having an asymmetric tooth root radius of curvature.

The calculated values for the stress ratio and the volume ratio of the target gear pair are compared in Table 2 with the values calculated from the test results in order to show the effects of applying asymmetric gear tooth root geometry.

Table 2 Comparison of estimated and experimental results for calculating size effect accurately

	Volume reduction ratio [%]
Estimated	9.5
Experimental result	10

The results indicate that the differences between the two sets of values were small, thus verifying the effectiveness of asymmetric tooth root geometry for downsizing transmission gears.

5. Conclusion

A method of designing automotive transmission gears with an asymmetric tooth root radius of curvature was investigated taking into account the field loading frequency during real-world vehicle use. The aim of this design method is to enable automotive transmission gears to be downsized by improving tooth root breakage strength.

This design method makes it possible to reduce the gear size and weight by 10% compared with conventionally designed gears. CVT units are now being mass produced with gears having an asymmetric tooth root radius of curvature.

6. References

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(3) Fukanoki, K., “Practical application of asymmetric tooth root geometry for downsizing automotive transmission gears,” VDI International Conference on Gears 2022, September 12-14, 2022, Germany, pp. 197-213.

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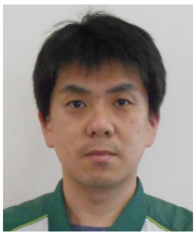
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